Simple Linear Regression R^2 and r Connection

BIOS 6611

CU Anschutz

Week 8

1 The Coefficient of Determination (R^2) and Correlation (r) Connection

The Coefficient of Determination (R^2) and Correlation (r) Connection

The Connection

In simple linear regression, the square root of R^2 is equivalent to the correlation coefficient between X and Y (i.e., $r_{x,y}$).

We will show this on the whiteboard starting with

$$R^2 = \frac{SS_{Model}}{SS_{Total}} = \frac{\sum_{i=1}^{n}\left(\hat{Y}_i - \bar{Y}\right)^2}{\sum_{i=1}^{n}\left(Y_i - \bar{Y}\right)^2} = \frac{\sum_{i=1}^{n}\left(\hat{\beta}_0 + \hat{\beta}_1 X_i - \bar{Y}\right)^2}{\sum_{i=1}^{n}\left(Y_i - \bar{Y}\right)^2}$$

The Implication

So what does the connection in simple linear regression of R^2 and r imply?

First, it implies that many of our concepts have connections that may not always seem apparent at first glance!

Second, if we were only interested in identifying if a predictor, X, is able to explain the amount of variance we observe in our outcome, Y, we could calculate the Pearson correlation and square its value. (Practically, this could be more efficient than looping through a bunch of SLRs.)